converges. The quantity $\eta\ (\alpha_3)$ becomes zero according to (3.12). From this we obtain that $k + r\varkappa$ grows without bounds for $\alpha \to \alpha_3$.

From (2.8) and (2.11) it follows that σ_{i3}, e_{i3}^p, v_3 also grow without bounds for $\alpha \to \alpha_3$.

Thus, if one considers only the bounded solutions in the packet of reflected waves, then a reflected plastic shock wave does not exist.

BIBLIOGRAPHY

1. Mandel, J., Ondes plastique dans un milieu indéfini à trois dimensions. J. Mechanique Vol.1, №1, 1962.
2. Bykovtsev, G. I., Ivlev, D. D. and Martynova, T. N., Propagation of waves in elastic-plastic bodies with piecewise linear conditions of plasticity. Proceedings of the All-Union Symposium on Propagation of Elastic-Plastic Waves in Continuous Media (Baku, October 1964). Izd. Akad. Nauk AzSSR, 1966.
3. Bykovtsev, G. I. and Kretova, L. D., Acceleration waves in ideal elastic-plastic bodies. Inzh. Zh. MTT, №1, 1967.
4. Thomas, T. Y., Plastic Flow and Fracture in Solids. New York, Academic Press, 1961.
5. Frank, Ph. and von Mises, R., Differential and Integral Equations of Mathematical Physics, Part 2, Chapter 12. Sobolev, S. L., Some Problems in the Theory of Wave Propagation. M.-L., ONTI, 1937.
6. Prager, V., Introduction to Mechanics of Continuous Media. M., (Russian Translation), Izd. Inostr. Lit., 1963.

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ON AN EFFECTIVE METHOD OF SOLVING NONCLASSICAL MIXED PROBLEMS OF THE THEORY OF ELASTICITY

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An integral equation of the first kind with a difference kernel having a logarithmic singularity is studied between finite limits. Many plane and three-dimensional mixed problems of elasticity theory and mathematical physics reduce to such integral equations.

A method is proposed for the effective solution of this equation for small values of the characteristic dimensionless parameter λ in the kernel. The principal part of the solution is extracted for small λ and the residual is sought in the form of some series of Laguerre polynomials. A certain infinite algebraic system is obtained to determine the coefficients of this series. An approximate solution of the integral equation with isolated characteristic singularities is found by truncating this system.

As illustrations, problems on the effect of a strip stamp on an elastic half-space and the impression of a stamp into an elastic strip are considered.

Certain papers of Popov [1-3] were the impetus to the development of this method.

1. Let us consider the integral equation

$$\int_{-1}^{1} q(\xi) M\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi f(x), \qquad |x| \leqslant 1 \tag{1.1}$$

Here $\lambda \in (0, \infty)$ is a dimensionless parameter.

The kernel of the equation is representable by the Fourier integral

$$M(y) = \int_0^\infty \frac{L(u)}{u} \cos uy\,du \tag{1.2}$$

Let us assume the following as regards the function $L(u)$.

1) $L(u) / u$ is an even, real, continuous function, and strictly positive for all $u \in (-\infty, \infty)$

$$2)\ L(u) = 1 + \frac{c_1}{|u|} + \frac{c_2}{u^2} + O(|u|^{-3}) \quad \text{for} \quad |u| \to \infty \tag{1.3}$$

$$3)\ L(u) \sim Au \quad \text{for} \quad u \to 0 \quad (A = \text{const} > 0)$$

Certain plane contact problems for a strip, wedge, a number of contact problems for cylindrical bodies, problems on the impression of a strip or annular stamp in an elastic half-space, etc., reduce to the integral equations of the type (1.1)–(1.3).

There holds [4–6] relative to the integral equation (1.1)–(1.3).

Theorem 1.1. Let the function $f(x)$ be such that $f'(x)$ satisfies the Hölder condition for $|x| \leqslant 1$ with exponent α, where $0 < \alpha < 1$. Then (1.1)–(1.3) is uniquely solvable in $L_p(-1, 1)$, $1 < p < 2$ for all $\lambda \in (0, \infty)$ and its solution is

$$q(\xi) = (1 - \xi^2)^{-1/2} Q(\xi) \tag{1.4}$$

where the function $Q(\xi)$ is at least continuous for $|\xi| \leqslant 1$.

Furthermore, taking account of the known theorem of Krein [7], let us limit ourselves to the analysis of the case $f(x) \equiv f$.

Theorem 1.2. If for some $\lambda \in (0, \infty)$, the function $\omega(\beta) \in L_p(0, \infty)$, $1 < p < 2$ is a solution of the integral equation

$$\int_0^\infty \omega(\beta) M(\beta - b)\,d\beta = \int_{2/\lambda}^\infty \left[\omega(\beta) + \frac{f}{A\lambda}\right] M\left(\beta + b - \frac{2}{\lambda}\right) d\beta \quad (0 \leqslant b < \infty) \tag{1.5}$$

then the solution of the integral equation (1.1) for $f(x) \equiv f$ in the class $L_p(-1, 1)$ is found by means of the formula

$$q(x) = \omega\left(\frac{1+x}{\lambda}\right) + \omega\left(\frac{1-x}{\lambda}\right) + \frac{f}{A\lambda}, \qquad |x| \leqslant 1 \tag{1.6}$$

Conversely, if $q(x) \in L_p(-1, 1)$, $1 < p < 2$ is a solution of (1.1) for $f(x) \equiv f$ and some $\lambda \in (0, \infty)$, then the function $\omega(b)$ defined in conformity with (1.6), is a solution of (1.5) in $L_p(0, \infty)$.

Proof. Let us consider the system of three integral equations [8]:

$$\int_{-\infty}^{\infty} v(\xi) M\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi f \qquad (-\infty < x < \infty) \tag{1.7}$$

$$\int_{-1}^{\infty} \omega\left(\frac{1+\xi}{\lambda}\right) M\left(\frac{\xi - x}{\lambda}\right) d\xi = \int_{-\infty}^{-1} \left[\omega\left(\frac{1-\xi}{\lambda}\right) + v(\xi)\right] M\left(\frac{\xi - x}{\lambda}\right) d\xi \ (-1 \leqslant x < \infty)$$

$$\int_{-\infty}^{1} \omega\left(\frac{1-\xi}{\lambda}\right) M\left(\frac{\xi - x}{\lambda}\right) d\xi = \int_{1}^{\infty} \left[\omega\left(\frac{1+\xi}{\lambda}\right) + v(\xi)\right] M\left(\frac{\xi - x}{\lambda}\right) d\xi \ (-\infty < x \leqslant 1)$$

By adding (1.7) it is easy to see that if the solution of this system is known, the solution of (1.1) has the form

$$q(x) = \omega\left(\frac{1+x}{\lambda}\right) + \omega\left(\frac{1-x}{\lambda}\right) + v(x), \qquad |x| \leqslant 1 \tag{1.8}$$

There still remains to take into account that the first equation in (1.7) has the solution

$$v(x) = f(A\lambda)^{-1} \tag{1.9}$$

and the second and third equations in (1.7) reduce to (1.5) by obvious changes of variables. The converse assertion of the theorem follows at once from the uniqueness of the solution of (1.1).

Corollary 1.1. Equation (1.5) is uniquely solvable in $L_p(0, \infty)$ for all $\lambda \in (0, \infty)$.

2. In constructing effective solutions of the integral equation (1.1), the idea of utilizing the smallness or largeness of the parameter λ naturally arises. This results in the idea of utilizing asymptotic methods [8] to investigate (1.1).

For ladge λ the asymptotic methods are based on the extraction and exact inversion of the integral operator corresponding to the logarithmic part [6] of the kernel $M(y)$. Namely, (1.1) is written as

$$-\int_{-1}^{1} q(\xi)\ln\left|\frac{\xi-x}{\lambda}\right| d\xi = \pi f(x) + \int_{-1}^{1} q(\xi) F\left(\frac{\xi-x}{\lambda}\right) d\xi, \qquad |x| \leqslant 1 \tag{2.1}$$

where the function $F(y)$ is the regular part of the kernel for all $\lambda > 0$; it can be shown [6] that under the conditions (1.3) it is at least continuous on the segment $|y| \leqslant 2/\lambda$. Furthermore, the integral equation (1.5) is solved either by successive approximations according to the scheme

$$-\int_{-1}^{1} q_i(\xi)\left[\ln\left|\frac{\xi-x}{\lambda}\right| - F(0)\right] d\xi = \pi f(x) + \int_{-1}^{1} q_{i-1}(\xi)\left[F\left(\frac{\xi-x}{\lambda}\right) + F(0)\right] d\xi$$

$$|x| \leqslant 1 \tag{2.2}$$

or is reduced to an infinite algebraic system. To do this the solution $q(\xi)$ is represented as

$$q(\xi) = \sum_{n=0}^{\infty} S_n T_n(\xi)(1-\xi^2)^{-1/2} \tag{2.3}$$

and the fact is utilized that the Chebyshev polynomials $T_n(\xi)$ are eigenfunctions of the operator

$$B(Q) = -\frac{1}{\pi}\int_{-1}^{1} \frac{Q(\xi)}{\sqrt{1-\xi^2}} \ln|\xi - x|\, d\xi \tag{2.4}$$

Asymptotic methods for small λ are based on the utilization of the equivalent equation (1.5) in place of (1.1) and the exact inversion of the Wiener-Hopf operator on the left. The specific solution of (1.5) can be obtained by successive approximations by means of the scheme

$$\int_{0}^{\infty} \omega_i(\beta) M(\beta - b)\, d\beta = \int_{2/\lambda}^{\infty} \left[\omega_{i-1}(\beta) + \frac{f}{A\lambda}\right] M\left(\beta + b - \frac{2}{\lambda}\right) d\beta, \quad 0 \leqslant b < \infty \tag{2.5}$$

or by reducing the integral equation (1.5) to an infinite algebraic system [5, 8, 9]. The method of reduction to an infinite system, developed in [5, 9], does not directly afford the possibility of obtaining a solution with the characteristic singularity $(1-x^2)^{-1/2}$

extracted at once (see (1.4)). A new method of reducing (1.5) to an infinite system will be given below which is based on knowing the eigenfunctions of some Wiener-Hopf integral operator, and permits obtaining the solution at once in the form (1.4).

We shall later need some results from papers [1, 3] where the validity of the following relationships is shown:

$$\int_0^\infty \frac{e^{-\tau}}{\sqrt{\tau}} L_m^{-1/2}(2\tau) K_0(\tau - t)\, d\tau = \frac{\pi}{\gamma_m} e^{-t} L_m^{-1/2}(2t) \tag{2.6}$$

$$\int_0^\infty \frac{U_{2m}(\sqrt{1-e^{-\tau\pi}})}{\sqrt{1-e^{-\tau\pi}}} e^{-\tau\pi} k_0(\tau - t)\, d\tau = \frac{e^{-\pi t/2} U_{2m}(1-e^{-\pi t})^{1/2}}{1/2 + m} \tag{2.7}$$

Here $K_0(z)$ is the Macdonald function, $L_m^\lambda(x)$ are Laguerre polynomials, $U_m(x)$ Chebyshev polynomials of the second kind

$$k_0(z) = -\ln|\operatorname{th}(\pi z/4)|, \qquad \gamma_m = \sqrt{2/\pi}\,(2m)!!\,[(2m-1)!!]^{-1} \tag{2.8}$$

3. Let us represent the integral equation (1.5) as

$$\int_0^\infty \varphi(\tau) K_*(\tau - t)\, d\tau = -\int_0^\infty \varphi(\tau) N(\tau - t)\, d\tau + \tag{3.1}$$

$$+ \int_0^\infty \left[\varphi(\tau + s) + \frac{1}{2} fs\right] K(\tau + t)\, d\tau \qquad (0 \leqslant t < \infty)$$

Here we use the notation

$$\frac{\beta}{A} = \tau, \quad \frac{b}{A} = t, \quad \frac{2}{A\lambda} = s, \quad M(y) = K\left(\frac{y}{A}\right), \quad \omega(A\tau) = \varphi(\tau)$$

$$K(z) = K_*(z) + N(z) = \int_0^\infty \frac{L(w/A)}{w} \cos wz\, dw \tag{3.2}$$

and $K_0(z)$ or $k_0(z)$ can be taken equally successfully as $K_*(z)$. The essence of the method proposed below will not thereby be altered. Furthermore, for definiteness, let us assume $K_*(z) = K_0(z)$.

Let us seek the solution of (3.1) in the form

$$\varphi(\tau) = \varphi_0(\tau) + \varphi_1(\tau) \tag{3.3}$$

where $\varphi_0(\tau)$ and $\varphi_1(\tau)$ are determined, respectively, from the integral equations

$$\int_0^\infty \varphi_0(\tau) K_0(\tau - t)\, d\tau = \frac{1}{2} fs \int_0^\infty K_0(\tau + t)\, d\tau \qquad (0 \leqslant t < \infty) \tag{3.4}$$

$$\int_0^\infty \varphi_1(\tau) K_0(\tau - t)\, d\tau = -\int_0^\infty [\varphi_0(\tau) + \varphi_1(\tau)] N(\tau - t)\, d\tau + \tag{3.5}$$

$$+ \frac{1}{2} fs \int_0^\infty N(\tau + t)\, d\tau + \int_0^\infty [\varphi_0(\tau + s) + \varphi_1(\tau + s)]\, K(\tau + t)\, d\tau \qquad (0 \leqslant t < \infty)$$

The solution of (3.4) can be obtained by the Wiener-Hopf method [10] and is

$$\varphi_0(t) = 0.5\, fs\, [\Phi(\sqrt{t}) + (\pi t)^{-1/2} e^{-t} - 1] \tag{3.6}$$

Here $\Phi(x)$ is the probability integral.

Let us expand $\varphi_0(t)$ in a series of Laguerre polynomials

$$\varphi_0(t) = \frac{1}{2} fs \frac{e^{-t}}{\sqrt{t}} \sum_{m=0}^{\infty} A_m L_m^{-1/2}(2t) \tag{3.7}$$

where the coefficients A_m are representable as

$$A_m = \gamma_m [(-1)^m \sqrt{2} - a_m], \quad a_m = \sqrt{2/\pi}\, \gamma_m^{-1} F(-m, 1; {}^1/_2; 2) \tag{3.8}$$

Here $F(\alpha, \beta; \gamma; \delta)$ is the hypergeometric function, and the constants a_m can also be found from the following recursion relations:

$$a_m + a_{m+1} = -(2m-1)!!\,[(2m+2)!!]^{-1} \qquad (m = 0, 1, \ldots;\ a_0 = 1)$$

Now let us expand the functions $N(\tau - t)$, $N(\tau + t)$, $K_0(\tau + t)$ in series of Laguerre polynomials. We obtain

$$N(\tau \pm t) = \sum_{m=0}^{\infty} \gamma_m b_m^{\pm}(\tau) e^{-t} L_m^{-1/2}(2t) \tag{3.9}$$

$$K_0(\tau + t) = \pi \sum_{m=0}^{\infty} d_m(\tau) e^{-t} L_m^{-1/2}(2t) \tag{3.10}$$

$$b_m^{\pm}(\tau) = \int_0^{\infty} N(\tau \pm t) \frac{e^{-t}}{\sqrt{t}} L_m^{-1/2}(2t)\, dt \tag{3.11}$$

The functions $d_m(\tau)$ $(m = 0, 1, \ldots)$ have the form [1]

$$d_m(\tau) = \frac{\sqrt{2}}{\pi} e^{-\tau} \int_0^{\infty} \frac{t^{m-1/2} e^{-t\tau}}{(t+2)^{1+m}}\, dt \tag{3.12}$$

Evaluating the integral [11] in (3.12), we obtain

$$d_m(\tau) = \sqrt{2/\pi}\,(2m-1)!!\, D_{-2m-1}(2\sqrt{\tau}) \qquad (m = 0, 1, \ldots) \tag{3.13}$$

Here $D_n(x)$ are parabolic cylinder functions. For n a negative integer, $D_n(x)$ are expressed [11] in terms of the function $\mathrm{E\,rfc}\,(x / \sqrt{2})$. It can be shown that $d_m(\tau)$ should satisfy the relationship

$$\int_0^{\infty} d_m(\tau)\, d\tau = (-1)^m \sqrt{2} - a_m \quad (m = 0, 1, \ldots) \tag{3.14}$$

Let us seek the function $\varphi_1(t)$ in a form analogous to (3.7)

$$\varphi_1(t) = \frac{1}{2} fs \frac{e^{-t}}{\sqrt{t}} \sum_{m=0}^{\infty} B_m L_m^{-1/2}(2t) \tag{3.15}$$

Inserting (3.7), (3.9), (3.10) and (3.15) into (3.5) and utilizing (2.6), we obtain the following infinite system to determine the B_m:

$$B_m = -C_m + \sum_{n=0}^{\infty} (A_n + B_n)(-R_{nm} + H_{nm} + M_{nm}) \quad (m = 0, 1, \ldots) \tag{3.16}$$

Here we have introduced the notation

$$C_m = \frac{\gamma_m^2}{\pi}\int_0^\infty b_m^-(\tau)\,d\tau, \quad R_{nm} = \frac{\gamma_m^2}{\pi}\int_0^\infty b_m^-(\tau)\frac{e^{-\tau}}{\sqrt{\tau}}L_n^{-1/2}(2\tau)\,d\tau$$

$$H_{nm} = e^{-s}\gamma_m\int_0^\infty \frac{e^{-\tau}d_m(\tau)}{\sqrt{\tau+s}}L_n^{-1/2}[2(\tau+s)]\,d\tau \tag{3.17}$$

$$M_{nm} = \frac{e^{-s}\gamma_m^2}{\pi}\int_0^\infty \frac{e^{-\tau}b_m^+(\tau)}{\sqrt{\tau+s}}L_n^{-1/2}[2(\tau+s)]\,d\tau \quad (m, n = 0, 1, \ldots)$$

Utilizing the integral 7.414(8) in [11] and the known relationships

$$\int_0^\infty \cos ux dx = \pi\delta(u), \quad \int_0^\infty \sin uxdx = \frac{1}{u} \quad (\delta(u) \text{ -- is delta function})$$

the constants C_m and R_{nm} can be represented in the following form convenient for calculations:

$$C_m = \frac{\sqrt{2}(-1)^m\gamma_m}{\pi}\int_0^\infty \frac{\varkappa(u)}{u(1+u^2)^{1/4}}\sin\left[\left(2m+\frac{1}{2}\right)\psi\right]du$$

$$R_{nm} = \frac{2(-1)^{n+m}\gamma_m}{\pi\gamma_n}\int_0^\infty \frac{\varkappa(u)}{(1+u^2)^{1/2}}\cos[2(n-m)\psi]\,du \tag{3.18}$$

$$\varkappa(u) = \frac{1}{u}L\left(\frac{u}{A}\right) - \frac{1}{\sqrt{u^2+1}}, \quad \psi = \operatorname{arctg} u, \quad n, m = 0, 1, \ldots$$

Taking account of (3.13), the constants H_{nm} can be evaluated by using integration by parts and 3.364(3) in [11]. Let us present the expressions defining these constants for some values of n and m

$$H_{00} = \frac{2}{\pi}s[K_0(s) + K_1(s)] - 2\left(\frac{2s}{\pi}\right)^{1/2}e^{-s}$$

$$H_{10} = \frac{s^2}{3\pi}\left[\left(\frac{3}{s} - 4\right)K_0(s) + \left(\frac{1}{s} - 4\right)K_1(s)\right] + \frac{s}{3}\left(\frac{2s}{\pi}\right)^{1/2}\left(4 - \frac{3}{s}\right)e^{-s}$$

$$H_{01} = 4H_{10} + 2\left(\frac{2s}{\pi}\right)^{1/2}e^{-s} \tag{3.19}$$

$$H_{11} = \frac{2s^3}{15\pi}\left[\left(\frac{15}{s^2} - \frac{36}{s} + 16\right)K_0(s) + \left(\frac{3}{s^2} - \frac{28}{s} + 16\right)K_1(s)\right] + \\ + \frac{s^2}{15}\left(\frac{2s}{\pi}\right)^{1/2}\left(-\frac{15}{s^2} + \frac{60}{s} - 32\right)e^{-s}$$

Here $K_\nu(s)$ are the Macdonald functions. Taking account of (3.11) the constants M_{nm} can be transformed into

$$M_{nm} = \frac{(-1)^m\sqrt{2}\gamma_m}{\pi}\int_0^\infty \frac{\varkappa(u)}{(1+u^2)^{1/4}}\chi_{nm}(u)\,du \tag{3.20}$$

$$\chi_{nm}(u) = e^{-s}\operatorname{Re}\left\{e^{-i(2m+1/2)\psi}\int_0^\infty \frac{e^{-\tau(1+iu)}}{\sqrt{\tau+s}}L_n^{-1/2}[2(\tau+s)]\,d\tau\right\}$$

The functions $\chi_{nm}(u)$ can be evaluated by using formulas 3.362(2) in [11]. For example

$$\chi_{0m}(u) = \frac{2}{(1+u^2)^{1/4}} \operatorname{Re}\{e^{-[(2m+1)\psi+iu]i} \operatorname{Erfc}(\sqrt{s(1+iu)})\} \quad (m = 0, 1, \ldots) \quad (3.21)$$

Let us now turn to the question of solving the system (3.16). As computations show, for $\lambda \leqslant 1$ terms containing the unknown coefficients B_n in the right side of the system can be neglected with adequate accuracy for practical purposes. Afterwards, the sum in the right side can be represented as an integral on the basis of the relationships (3.6), (3.7). When $\lambda \leqslant 2$, terms containing M_{nm} in the right side can be neglected in solving the system. After the solution of the system, the function $q(x)$ is found by means of (1.6), (3.2), (3.3), (3.6), and (3.15). The following expression

$$P = f\left[P_0 + 2\sum_{m=0}^{\infty} B_m \int_0^s \frac{e^{-\tau}}{\sqrt{\tau}} L_m^{-1/2}(2\tau)\, d\tau\right]$$

$$P_0 = (2s+1)\Phi(\sqrt{s}) - s + 2\sqrt{s/\pi}e^{-s} \quad (3.22)$$

can be obtained for the integral characteristic of the solution

$$P = \int_{-1}^{1} q(x)\, dx$$

4. As illustrations, let us consider the following contact problem of elasticity theory: (a) the problem of the impression of a smooth strip stamp in a half-space [1, 4, 8, 12], and (b) the problem of the impression of a stamp in a strip lying frictionless on a rigid foundation [13]. For the problems (a) and (b), respectively, the function $L(u)$ is

$$\text{(a)} \quad L(u) = \frac{u}{\sqrt{u^2+1}}, \qquad \text{(b)} \quad L(u) = \frac{\operatorname{ch} 2u - 1}{\operatorname{sh} 2u + 2u} \quad (4.1)$$

Let us solve the system (3.16) by the truncation method. Keeping a finite number of terms in the series (3.16) and solving the system thus found, we obtain an approximate solution of the problem. As numerical computations show, in order to obtain a practically exact solution for $\lambda \leqslant 2$, it is sufficient to limit oneself to the solution of a system of two or three equations.

For the problems (a) and (b) being considered, the solutions of the truncated systems for $\lambda = 2$ comprised of one, two, and three equations, respectively, are

(a) (1) $B_0 = 0.0212$, (2) $B_0 = 0.00210$, $B_1 = 0.00212$
(3) $B_0 = 0.00328$, $B_1 = 0.00171$, $B_2 = 0.00121$
(b) (1) $B_0 = -0.0431$ (2) $B_0 = -0.0556$, $B_1 = 0.0800$
(3) $B_0 = -0.0471$, $B_1 = 0.0828$, $B_2 = 0.00985$

From (3.22) we obtain an approximate expression to determine the force

$$P = f\{P_0 + 2\sqrt{\pi}\,\Phi(\sqrt{s})\, B_0 + [4\sqrt{s}\,e^{-s} - \sqrt{\pi}\,\Phi(\sqrt{s})]\, B_1 + [{}^3/_4\sqrt{\pi}\,\Phi(\sqrt{s}) - 4s\sqrt{s}\,e^{-s}]\, B_2\} \quad (4.2)$$

Let us present the values of the quantities $P_* = f^{-1}P$, $Q_1 = f^{-1}Q(1)$ and $Q_2 = f^{-1}Q(0)$ (the first, second, and third approximations), calculated for $\lambda = 2$ for the problems (a) and (b) by the formulas in the present paper and the formulas obtained by the method of large λ [8].

	P_*		Q_1		Q_2	
	(a)	(b)	(a)	(b)	(a)	(b)
(1)	2.00	2.84	0.585	0.737	0.685	1.069
(2)	1.95	2.73	0.567	0.776	0.668	0.971
(3)	1.95	2.75	0.569	0.795	0.668	0.970
[8]	1.96	2.75	0.580	0.795	0.669	0.964

In conclusion, let us note that the mentioned method can be utilized successfully to investigate problems on the impression of a sufficiently broad ring stamp in an elastic half-space if it is taken into account that the eigenfunctions of the Wiener-Hopf integral operator

$$A_*\varphi = \frac{2}{\pi}\int_0^\infty \varphi(\tau)\,d\tau \int_0^\infty \frac{\Gamma(1/4 + 1/2 iu)\,\Gamma(1/4 - 1/2 iu)}{\Gamma(3/4 + 1/2 iu)\,\Gamma(3/4 - 1/2 iu)} \cos u(\tau - t)\,du \tag{4.3}$$

are known [2].

BIBLIOGRAPHY

1. Popov, G. Ia., On an approximate method of solving the integral equation of electromagnetic wave diffraction by a strip. Zh.Tekh.Fiz., Vol. 35, №3, 1965.
2. Popov, G. Ia., On an approximate method of solving the contact problem of a ring stamp. Izv. Akad. Nauk ArmSSR, Mekhanika, Vol. 20, №2, 1967.
3. Lutchenko, S. A. and Popov, G. Ia., On some plane contact problems of the theory of elasticity for a wedge. Prikl. Mekh., Vol. 6, №3, 1970.
4. Babeshko, V. A., Asymptotic properties of the solution of a class of integral equations originating in elasticity theory and mathematical physics. Dokl. Akad. Nauk SSSR, Vol. 186, №6, 1969.
5. Babeshko, V. A., Certain type of integral equations appearing in contact problems of the theory of elasticity. PMM Vol. 33, №6, 1969.
6. Aleksandrov, V. M. and Belokon', A. V., Asymptotic solution of a class of integral equations encountered in the investigation of mixed problems of mathematical physics for regions with cylindrical boundaries. PMM, Vol. 32, №3, 1968.
7. Krein, M. G., On a new method of solving linear integral equations of the first and second kind. Dokl. Akad. Nauk SSSR, Vol. 100, №3, 1955.
8. Aleksandrov, V. M., Asymptotic methods in contact problems of elasticity theory. PMM Vol. 32, №4, 1968.
9. Babeshko, V. A., On an asymptotic method to the solution of integral equations in the theory of elasticity and in mathematical physics. PMM Vol. 30, №4, 1966.
10. Noble, B., Application of the Wiener-Hopf Method to Solve Partial Differential Equations. Moscow, IIL, 1962.
11. Gradshtein, I. S. and Ryzhik, Tables of Integrals, Sums, Series and Products. Moscow, Fizmatgiz, 1963.
12. Rvachev, V. L., The pressure of a punch having the platform of a strip, upon an elastic half-space. PMM Vol. 20, №2, 1956.
13. Aleksandrov, V. M., On the approximate solution of a certain type of integral equation. PMM Vol. 26, №5, 1962.

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